# Coherent Structures

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In order to develop more quantitative measures of coherent structures that would have comparative value over a range of experiments, it is essential that such measures be *independent of the observer*. It is only through such a general framework that theories with a fundamental predictive value can be developed. The triple decomposition

$$\phi = \bar{\phi} + \phi_c + \phi_r \tag{1}$$

(where $\bar{\phi}$ is the mean, $\phi_c$ is the coherent part, and $\phi_r$ is the random part of any turbulent field $\phi$) serves this purpose. For a statistically steady turbulence which possesses coherent structures with a dominant temporal frequency $f$ we can take (see Hussain 1983)

$$\bar{\phi} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \phi(\mathbf{x},t)dt \tag{2}$$

$$\phi_c = <\phi'> \equiv \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\phi'(\mathbf{x},t+t_i) \tag{3}$$

where $\phi' \equiv \phi - \bar{\phi}, t_i = i/f$, and $< \cdot >$ denotes the phase average. For more general turbulent flows (that are not necessarily statistically steady or do not possess coherent structures with a dominant temporal frequency), on can take

$$\bar{\phi} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\phi^{(i)}(\mathbf{x},t) \tag{4}$$

where an ensemble average is taken over $N$ repeated experiments with the same initial and boundary conditions. The coherent part of the turbulence can be taken to be

$$\phi_c = <\phi'|E> \tag{5}$$

where $< \cdot|E >$ denotes a suitable conditional average of $\phi'$ (i.e., an ensemble average over flow structures subject to the occurrence of some event $E$. With such triple decompositions, the coherent and random parts of the turbulence will be the same for all observers (see Speziale 1986).

It should be noted that double decompositions (see Hussain 1983, 1986) give rise to coherent and random parts of any turbulent field $\phi$ that, in general, depend on the observer. Double decompositions should therefore only be used when the mean flow vanishes or is negligibly small compared to the coherent notion (see Speziale 1986). Otherwise, one runs the risk of extracting flow structures that are overly biased by the observer.

The equations of motion for the mean and coherent flow fields, based on the triple decomposition (1), can be written in the form (see Hussain 1983):

$$\frac{D\bar{u}_i}{Dt} = -\frac{\partial\bar{p}}{\partial x_i} + \nu\nabla^2\bar{u}_i - \frac{\partial}{\partial x_j}(\overline{u_{c_i}u_{c_j}} + \overline{u_{r_i}u_{r_j}}) \tag{6}$$

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$$\frac{Du_{c_i}}{Dt} = -\frac{\partial p_c}{\partial x_i} + \nu\nabla^2 u_{c_i} - u_{c_j}\frac{\partial \bar{u}_i}{\partial x_j} - u_{c_j}\frac{\partial u_{c_i}}{\partial x_j} \\ -\frac{D\bar{u}_i}{Dt} - \frac{\partial \bar{p}}{\partial x_i} + \nu\nabla^2\bar{u}_i - \frac{\partial}{\partial x_j} < u_{r_i}u_{r_j} > \tag{7}$$

where $\nu$ is the kinematic viscosity of the fluid, $p$ is the modified pressure, and $u = \bar{u} + u_c + u_r$ is the velocity field which is subject to the continuity equation which yields the constraints

$$\nabla \cdot \bar{\mathbf{u}} = 0 \tag{8}$$

$$\nabla \cdot \bar{\mathbf{u}}_c = 0 \tag{9}$$

$$\nabla \cdot \bar{\mathbf{u}}_r = 0 \tag{10}$$

In order to achieve closure of the equations of motion (6)- (10), the Reynolds stress terms

$$< u_{r_i}u_{r_j} >, \overline{u_{r_i}u_{r_j}} \tag{11}$$

need to be modeled. The time-averaged Reynolds stress $\overline{u_{r_i}u_{r_j}}$ can be modeled using the currently popular two-equation models or second-order closure models (see Launder, Reece and Rodi 1975 and Lumley 1978). The phase-averaged Reynolds stress $< u_{r_i}u_{r_j} >$ primarily serves as an energy drain on the coherent motion and thus it is plausible that it could be modeled using a gradient transport hypothesis (see Hussain 1983). Hence, eddy viscosity models of the form

$$< u_{r_i}u_{r_j} >= -\nu_T\left(\frac{\partial u_{c_i}}{\partial x_j} + \frac{\partial u_{c_j}}{\partial x_i}\right) \tag{12}$$

can be considered where $\nu_T$ is an appropriate eddy viscosity (sufficiently far from solid boundaries, the Smagorinsky model can be tried). This approach, which bears a certain resemblance to large-eddy simulations, has an advantage in that the coherent motion $\mathbf{u}_c$ is calculated directly. Furthermore, the level of computation required is substantially less than that needed for a direct numerical simulation since a coarse mesh can be used (the fine-scale turbulence is modeled) and for some problems (e.g., turbulent mixing layers) the coherent motion is approximately two-dimensional. In my opinion, there is a good chance that such an approach could yield useful new information concerning the nature of coherent structures and it is well worth pursuing future investigations along these lines.

## REFERENCES

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